Optimising Typed Programs – Exercises

Note: There is an error in the notes; you need to replace the requirement $\vec{\alpha} = \text{ftv}(\tau) \setminus \text{ftv}(\Gamma)$ in rules (6) and (9) with the requirement $\text{ftv}(\vec{\alpha}) \cap \text{ftv}(\Gamma) = \emptyset$. Otherwise, Lemma 2.1 does not hold.

Exercise 1 Give an example showing that simultaneous substitution is different from compositional substitution, i.e., show that there exist types $\tau_1$, $\tau_2$ and $\tau$, such that

$\tau\{\alpha_1 \mapsto \tau_1, \alpha_2 \mapsto \tau_2\} \neq (\tau\{\alpha_1 \mapsto \tau_1\}\{\alpha_2 \mapsto \tau_2\})$

Exercise 2 Prove Lemma 2.1.

Exercise 3 Show that $\rightarrow_{\text{proj}}$ is type preserving.

Exercise 4 Show that $\rightarrow_{\text{dce}}$ is type preserving.

Exercise 5 Assume that the typed lambda language is extended to support integers. What is the result of applying $\rightarrow_{\text{spec2}}$ and $\rightarrow_{\text{int1}}$ to the expression

```latex
\begin{align*}
\text{letrec } adder &: \text{int} \rightarrow \text{int} \rightarrow \text{int} \\
&= \lambda n : \text{int}. \ if \ n = 0 \ then \ 0 \\
&\quad \ else \ g \ n + adder \ g \ (n - 1) \\
&\quad \ in \ adder \ (\lambda x : \text{int}. x + 1) \ 10
\end{align*}
```

Exercise 6 What is the result of applying value propagation to the expression

```latex
\begin{align*}
\lambda y : \text{bool}. \lambda x : \text{bool}.
& \ if \ x \ then \\
& \quad \ if \ x \ then \ y \ else \ true \\
& \quad \ else \ y
\end{align*}
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